

Chapter 14. Statistic

Question-1

The following table shows marks secured by 140 students in an examination:

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Calculate mean marks by using all the three methods, i.e., direct method, assumed mean method and step-deviation method.

Solution:

Direct Method

Marks	Number of Students (f_i)	Mid-point (x_i)	$f_i x_i$
0 - 10	20	5	100
10 - 20	24	15	360
20 - 30	40	25	1000
30 - 40	36	35	1260
40 - 50	20	45	900
Total	140		3620

Hence mean $\bar{x} = \frac{1}{n} \sum_{i=1}^5 f_i x_i = \frac{3620}{140} = 25.86$ marks

Assumed Mean Method

Marks	Number of Students (f_i)	Mid-point (x_i)	Deviation $d_i = x_i - 25$	$f_i d_i$
0 - 10	20	5	-20	-400
10 - 20	24	15	-10	-240
20 - 30	40	$a = 25$	0	0
30 - 40	36	35	10	360
40 - 50	20	45	20	400
Total	140			120

Hence mean $\bar{x} = a + \frac{1}{n} \sum_{i=1}^5 f_i d_i = 25 + \frac{120}{140} = 25.86$ marks

Step-Deviation Method

Marks	Number of Students (f_i)	Mid-point (x_i)	Deviation $u_i = (x_i - 25)/10$	$f_i u_i$
0-10	20	5	-2	-40
10 - 20	24	15	-1	-24
20 - 30	40	$a=25$	0	0
30 - 40	36	35	1	36
40 - 50	20	45	2	40
Total	140			12

Hence mean $\bar{x} = a + h \times \frac{1}{n} \sum_{i=1}^5 f_i u_i$ [where $h = 10$]

$$= 25 + 10 \times \frac{12}{140} = 25.86 \text{ marks}$$

Question-2

Find the mean age in years from the frequency distribution given below:

Class Interval of Age in years	Frequency f_i
25 - 29	4
30 - 34	14
35 - 39	22
40 - 44	16
45 - 49	6
50 - 54	5
55 - 59	3
Total	70

Solution

Class Interval of Age in years	Frequency (f_i)	Mid-point (x_i)	Deviation $u_i = (x_i - 42)/5$	$f_i u_i$
25 - 29	4	27	-3	-12
30 - 34	14	32	-2	-28
35 - 39	22	37	-1	-22
40 - 44	16	$a = 42$	0	0
45 - 49	6	47	1	6
50 - 54	5	52	2	10
55 - 59	3	57	3	9
Total	70			-37

Hence mean $\bar{x} = a + h \times \frac{1}{n} \sum_{i=1}^7 f_i u_i$ [where $h = 5$]

$$= 42 + 5 \times \frac{-37}{70} = 39.36 \text{ years}$$

Question-3

New Page 2 Find the mean marks from the following data:

Marks	Number of Students
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

Solution:

Marks	Number of Students (f_i)	Mid-point (x_i)	Deviation $u_i = (x_i - 55)/10$	$f_i u_i$
0 - 10	5	5	-5	-25
10 - 20	9 - 5 = 4	15	-4	-16
20 - 30	17 - 9 = 8	25	-3	-24
30 - 40	29 - 17 = 12	35	-2	-24
40 - 50	45 - 29 = 16	45	-1	-16
50 - 60	60 - 45 = 15	a = 55	0	0
60 - 70	70 - 60 = 10	65	1	10
70 - 80	78 - 70 = 8	75	2	16
80 - 90	83 - 78 = 5	85	3	15
90 - 100	85 - 83 = 2	95	4	8
Total	85			-56

$$\begin{aligned}\text{Hence mean } \bar{x} &= a + h \times \frac{1}{n} \sum_{i=1}^{10} f_i u_i \quad [\text{where } h = 10] \\ &= 55 + 10 \times \frac{-56}{85} = 48.41 \text{ marks}\end{aligned}$$

Question-4

New Page 2 Find the mean age of 100 residents of a colony from the following data:

Age in years (Greater than or equal to)	Number of Persons
0	100
10	90
20	75
30	50
40	25
50	15
60	5
70	0

Solution:

Age	Number of Persons (f_i)	Mid-point (x_i)	Deviation $u_i = (x_i - 35)/10$	$f_i u_i$
0 - 10	$100 - 90 = 10$	5	-3	-30
10 - 20	$90 - 75 = 15$	15	-2	-30
20 - 30	$75 - 50 = 25$	25	-1	-25
30 - 40	$50 - 25 = 25$	$a = 35$	0	0
40 - 50	$25 - 15 = 10$	45	1	10
50 - 60	$15 - 5 = 10$	55	2	20
60 - 70	$5 - 0 = 5$	65	3	15
Total	100			-40

$$\begin{aligned}\text{Hence mean } \bar{x} &= a + h \times \frac{1}{n} \sum_{i=1}^7 f_i u_i \quad [\text{where } h = 10] \\ &= 35 + 10 \times \frac{-40}{100} = 31 \text{ years}\end{aligned}$$

Question-5

If the mean of the following data is 21.5, find the value of k:

x_i	5	15	25	35	45
f_i	6	4	3	k	2

Solution:

x_i	f_i	$x_i f_i$
5	6	30
15	4	60
25	3	75
35	k	35k
45	2	90
Total	$15 + k$	$255 + 35k$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$\frac{255 + 35k}{15 + k} = 21.5$$

$$21.5(15 + k) = 255 + 35k$$

$$21.5 \times 15 + 21.5k = 255 + 35k$$

$$35k - 21.5k = 322.5 - 255$$

$$13.5k = 67.5$$

$$\therefore k = \frac{67.5}{13.5} = 5$$

Question-6

Find the values of f_1 and f_2 of the frequency, if the mean of the following frequency distribution is 21.4 and the total frequency is 40.

Class-interval	0 - 8	8 - 16	16 - 24	24 - 32	32 - 40
Frequency	6	f_1	10	f_2	9

Solution:

Take $a = 10$ and $h = 8$.

Class interval	Mid-point x_i	f_i	$f_i x_i$
0 - 8	4	6	24
8 - 16	12	f_1	$12 f_1$
16 - 24	20	10	200
24 - 32	28	f_2	$28 f_2$
32 - 40	36	9	324
		$25 + f_1 + f_2 = 40$	$548 + 12 f_1 + 28 f_2$

$$f_1 + f_2 = 15 \dots\dots\dots(i)$$

$$\therefore \bar{x} = \frac{548 + 12f_1 + 28f_2}{40}$$

$$21.4 = \frac{548 + 12f_1 + 28f_2}{40}$$

$$856 = 548 + 12f_1 + 28f_2$$

$$12f_1 + 28f_2 = 308$$

$$3f_1 + 7f_2 = 77 \dots\dots\dots(ii)$$

$$(ii) - 3 \times (i) \Rightarrow$$

$$3f_1 + 7f_2 = 77$$

$$3f_1 + 3f_2 = 45$$

$$\begin{array}{r} \text{-----} \\ 4f_2 = 32 \end{array}$$

$$\begin{array}{r} \text{-----} \\ f_2 = 8 \therefore f_1 = 15 - 8 = 7 \end{array}$$

Question-7

The median of the observations 11, 12, 14, 18, $x + 2$, $x + 4$, 30, 32, 35, 41 arranged in ascending order is 24. Find the value of x .

Solution:

The number of observations = $n = 10$. Since n is even ,

$$\text{Median} = \frac{\left(\frac{n}{2}\right)\text{observation} + \left(\frac{n}{2} + 1\right)\text{observation}}{2}$$

$$\Rightarrow 24 = \frac{(5^{\text{th}})\text{observation} + (6^{\text{th}})\text{observation}}{2} = \frac{(x+2) + (x+4)}{2} = \frac{2x+6}{2}$$

$$\Rightarrow x = 21.$$

Question-8

Find the median of the following data: 19, 25, 59, 48, 35, 31, 30, 32, 51. If 25 is replaced by 52 and 19 by 29, what will be the new median?

Solution:

Arranging the data in ascending order: 19, 25, 30, 31, 32, 35, 48, 51, 59.

The number of observations $n = 9$ (odd)

$$\therefore \text{Median} = \left(\frac{9+1}{2}\right)\text{observation} = 5^{\text{th}}\text{observation} = 32.$$

Hence Median = 32.

If 25 is replaced by 52 and 19 by 29, the new observations in ascending order are :

29, 30, 31, 32, 35, 48, 51, 52, 59.

$$\therefore \text{New Median} = 5^{\text{th}}\text{observation} = 35.$$

Question-9

Find the value of p , if the mean of the following distribution is 20.

x	15	17	19	$20 + p$	23
f	2	3	4	$5p$	6

Solution:

x_i	f_i	$x_i f_i$
15	2	30
17	3	51
19	4	76
$20 + p$	$5p$	$100p + 5p^2$
23	6	138
Total	$15 + 5p$	$5p^2 + 100p + 295$

$$\text{Mean} = \bar{x} = \frac{1}{n}(\sum x_i)$$

$$20 = \frac{5p^2 + 100p + 295}{15 + 5p}$$

$$5p^2 + 100p + 295 = 300 + 100p$$

$$5p^2 - 5 = 0$$

$$5(p^2 - 1) = 0$$

$$(p + 1)(p - 1) = 0$$

$$p = 1 \text{ or } p = -1$$